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INFLUENCE OF AN ELLIPTICAL NON-UNIFORMITY OF DIELECTRIC SPHERE ON SPECTRAL CHARACTERISTICS OF RESONANCE OSCILLATIONS

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The spectral task is solved for open prolate spheroid that made from homogeneous isotropic crystal and has small distance between its focuses. The approximate dispersion equation is obtained at satisfaction of the tangential component continuity conditions of electromagnetic field strengths for resonance oscillations on the spheroid surface. It allows to study spectral characteristics of resonance oscillations.

INTRODUCTION

The open dielectric resonators are widely applied to determine the parameters of materials and to create of stable microwave standards and to use in the precision metering equipment. Thus, the measurement and research of spectral characteristics of resonance oscillations have important value. The rigorous solution of a set of Maxwell equations satisfying the radiations on infinity and the boundary conditions of electromagnetic field components was only determined for some limited dielectric structures. The numerical analysis of spectral characteristics made for the lowest types of resonance oscillations in an isotropic dielectric sphere [1]. The capability of excitation independent TE ($E_r = 0$) and TM ($H_r = 0$) wave modes was shown. In each of them there is a frequent degeneration at which the same frequency has $2n+1$ modes with various dependence from the azimuth coordinate φ. Here n parameter is polar index determining number of field variations on the polar coordinate θ. The influence of weak azimuthal non-uniformity leads to removing the degeneration and to arising independent EH and HE oscillations for which all six field components are not zero.
The resonance oscillations independent from azimuthal coordinate ($\partial / \partial\varphi \equiv 0$) were studied in open spheroidal structures [2-4]. The mention above effects does not arise in these approximations.

THEORETICAL CONSIDERATION

We consider an open spheroid made from homogeneous isotropic dielectric with permittivity ε_d and permeability μ_d. The prolate spheroidal coordinate system (ξ, η, φ) obtained by rotation of plane elliptical coordinate system around of a large axis (Figure 1) is used for the solution of the spectral task. In this system $\xi \in [0, 2\pi)$; $\eta \in [-1,1]$ and $\varphi \in [0, 2\pi]$, and the metrical Lame's coefficients are equal $h_\xi = \tau\sqrt{\dfrac{\xi^2 - \eta^2}{\xi^2 - 1}}$; $h_\eta = \tau\sqrt{\dfrac{\xi^2 - \eta^2}{1 - \eta^2}}$; $h_\varphi = \tau\sqrt{(\xi^2 - 1)(1 - \eta^2)}$, where τ is the distance

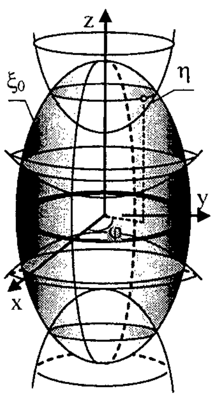

Figure 1.

between focal points. The surfaces $\xi = const$ are prolate spheroids in spheroidal coordinate system. The value $\eta = 0$ corresponds to crossing of the spheroid with a plane $z = 0$, and $\eta = \pm 1$ correspond to spheroid poles placed on the z axis [5]. At the coordinate $\xi = \xi_0$ it is limited by the medium with permittivity ε_b and permeability μ_b.

For fields proportionate to $\exp i(m\varphi - \omega_p t)$, where ω_p is the frequency of p-th mode oscillation and m is azimuthal index, the study of the set of Maxwell equations is reduced to the solution of two coupling differential equations about E_ξ and H_ξ components

$$i\varepsilon_j k\tau \Lambda_j h_\xi E_{j\xi} = -\frac{\partial}{\partial\varphi} L_j h_\xi H_{j\xi}\,;\; i\mu_j k\tau \Lambda_j h_\xi H_{j\xi} = \frac{\partial}{\partial\varphi} L_j h_\xi E_{j\xi}\,. \tag{1}$$

Here $k = \omega_p / c$, c is the light velocity and subscript j takes the value d inside or b outside of spheroid. The Λ_j and L_j operators are equal

$$\Lambda_j = \frac{\partial}{\partial\eta} g_{j\varphi}^{-1} \frac{\partial}{\partial\eta} + g_{j\eta}^{-1} \frac{\partial^2}{\partial\varphi^2} + \xi^2 - 1;\; L_j = \frac{\partial}{\partial\eta} g_{j\varphi}^{-1} \frac{1}{1-\eta^2} \frac{\partial}{\partial\xi} - g_{j\eta}^{-1} (1-\eta^2) \frac{\partial}{\partial\xi} \frac{\xi^2 - 1}{\xi^2 - \eta^2} \frac{\partial}{\partial\eta},$$

where $g_{j\eta}^{-1}$ and $g_{j\varphi}^{-1}$ operators are reverse for operators of

$$g_{j\eta} = (1-\eta^2)[a_j + \frac{\partial}{\partial\xi} \frac{\xi^2 - 1}{\xi^2 - \eta^2} \frac{\partial}{\partial\xi}];\; g_{j\varphi} = \frac{1}{(1-\eta^2)} [a_j \frac{\xi^2 - \eta^2}{\xi^2 - 1} + \frac{\partial^2}{\partial\xi^2}].$$

Let us consider an isotropic dielectric sphere with a small ellipticity along the z axis. The transformation to spherical coordinates (r, θ, φ) in (1) is carried out in the assumption $\varphi = \varphi$, $\eta = \cos\theta$ and tendency $\tau \to 0$, $\xi \to \infty$ so that the multiplication $\tau\xi \to r$ remained final [5]. We introduce potential functions U_j^S by the expression of $g_{j0} U_j^S = h_\xi S_{j\xi}$, where $g_{j0} = a_j + \partial^2 / \partial\xi^2$, $a_j = \varepsilon_j \mu_j k^2 \tau^2$ and S takes the value E or H. For large ξ values and $\eta = \cos\theta$ we obtain $g_{j\eta} \approx \sin^2\theta\, [g_{j0} - \frac{\sin^2\theta}{\xi^2} \frac{\partial^2}{\partial\xi^2}] + O(\xi^{-3})$, $g_{j\varphi} \approx \frac{g_{j0}}{\sin^2\theta} + \frac{a_j}{\xi^2} + O(\xi^{-3})$. The system (1) becomes

$$(\Lambda_{j0} - \Lambda_{j1}) U_j^E \approx -g_{j0}^{-1} \frac{2i\mu_j k\tau}{\xi^2} \cos\theta \frac{\partial^2 U_j^H}{\partial\varphi\, \partial\xi};$$

$$(\Lambda_{j0} - \Lambda_{j1}) U_j^H \approx g_{j0}^{-1} \frac{2i\varepsilon_j k\tau}{\xi^2} \cos\theta \frac{\partial^2 U_j^E}{\partial\varphi\, \partial\xi}, \tag{2}$$

where $\Lambda_{j0} = \Delta_\perp + \xi^2 g_{j0}$, $\Lambda_{j1} = g_{0j} + g_{j0}^{-1} \frac{1}{\xi^2} (\frac{a_j}{\sin\theta} \frac{\partial}{\partial\theta} \sin^3\theta \frac{\partial}{\partial\theta} - \frac{\partial^4}{\partial\varphi^2 \partial\xi^2})$,

$\Delta_\perp = \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \sin\theta \frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\varphi^2}$ and g_{j0}^{-1} operator is reverse for operator of g_{j0}.

At neglect by values of the order ξ^{-2} the system (2) breaks up to two independent subsystems describing TE and TM types of resonance oscillations in the spheroid. The resonance frequencies of them are accordingly determined by solving the equations

$$\sqrt{\varepsilon_d/\mu_d}\,\frac{j'_{\nu(n)}(x_d)}{j_{\nu(n)}(x_d)}=\sqrt{\varepsilon_b/\mu_b}\,\frac{h^{(1)\prime}_{\nu(n)}(x_b)}{h^{(1)}_{\nu(n)}(x_b)} \text{ и } \sqrt{\mu_d/\varepsilon_d}\,\frac{j'_{\nu(n)}(x_d)}{j_{\nu(n)}(x_d)}=\sqrt{\mu_b/\varepsilon_b}\,\frac{h^{(1)\prime}_{\nu(n)}(x_b)}{h^{(1)}_{\nu(n)}(x_b)}, \quad (3)$$

where $x_j=\sqrt{\varepsilon_j\mu_j}k\tau\xi_0$; $j_{\nu(n)}(x)=\sqrt{\pi x/2}J_{n+1/2}(x)$; $h^{(1)}_{\nu(n)}(x)=\sqrt{\pi x/2}H^{(1)}_{n+1/2}(x)$, $\nu(n)=n+1/2$, $J_{\nu(n)}(x)$ and $H^{(1)}_{\nu(n)}(x)$ are the Bessel and first-kind Hankel cylindrical functions of the n-th order.

In the equations (3) the azimuthal index m is absent. Hence each resonance oscillation mode is $2n+1$-fold degenerate.

The taking into consideration of the addends proportionate to ξ^{-2} in (2) allows to study the influence of ellipsoidal non-uniformity upon the parameters of resonance oscillations. The TM oscillations are transformed in the HE and TE are transformed in the EH oscillations for which all six field components are already not zero. In this case the solutions of system (2) are proportionate to functions of $j_{\nu(n,d)}(\sqrt{a_d}\xi)$ inside and $h^{(1)}_{\nu(n,b)}(\sqrt{a_b}\xi)$ outside of the spheroid, where $\nu(n,j)=[(n+1/2)^2+(m^2+\gamma_n)a_j/n(n+1)]^{1/2}$, $\gamma_n=f_{n-1}f_{n+1}[2n^2(n+1)^2-m^2(2n^2+2n+3)]$ and $f_n=1/(2n+1)$.

The resonance frequencies of oscillations are determined by solving the equations

$$\sqrt{\varepsilon_d/\mu_d}\,\frac{j'_{\nu(n,d)}(x_d)}{j_{\nu(n,d)}(x_d)}=\sqrt{\varepsilon_b/\mu_b}\,\frac{h^{(1)\prime}_{\nu(n,b)}(x_b)}{h^{(1)}_{\nu(n,b)}(x_b)}$$

for EH and

$$\sqrt{\mu_d/\varepsilon_d}\,\frac{j'_{\nu(n,d)}(x_d)}{j_{\nu(n,d)}(x_d)}=\sqrt{\mu_b/\varepsilon_b}\,\frac{h^{(1)\prime}_{\nu(n,b)}(x_b)}{h^{(1)}_{\nu(n,b)}(x_b)}$$

for HE types.

The dependence of $\nu(n,j)$ parameter from the azimuthal index m arising under influence of the ellipsoidal non-uniformity of dielectric sphere removes the frequency degeneration for resonance oscillations.

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